14.3 Videos Guide

14.3a

Definition: (partial derivative)

- $\frac{\partial z}{\partial x} = f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$
- $\frac{\partial z}{\partial y} = f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$

Exercise:

- Find the first partial derivatives of the function.
  $z = x\sin(xy)$

14.3b

- Higher-order partial derivatives
  - $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 y}{\partial y \partial x} = f_{xy}$ denotes the partial derivative first with respect to x and then with respect to y

Theorem (statement):

- The conclusion of Clairaut's Theorem is that second mixed partial derivatives are equal: $f_{xy} = f_{yx}$

Exercises:

- Verify that the conclusion of Clairaut's Theorem holds, that is, $u_{xy} = u_{yx}$.
  $u = e^{xy}\sin y$

14.3c

- Find the first partial derivatives of the function.
  $f(x,y,z) = xy^2e^{-xz}$
- Use implicit differentiation to find $\partial x/\partial z$ and $\partial x/\partial y$.
  $x^2 - y^2 + z^2 - 2z = 4$

14.3d

- Determine the signs of the partial derivatives for the function f whose graph is shown.
  (a) $f_{xy}(1,2)$ (b) $f_{xy}(-1,2)$